

1. [5.1 Introduction to Work, and Energy](#)
2. [5.2 Work: The Scientific Definition](#)
3. [5.3 Kinetic Energy and the Work-Energy Theorem](#)
4. [5.4 Gravitational Potential Energy](#)
5. [5.5 Conservative Forces and Potential Energy](#)
6. [5.6 Nonconservative Forces](#)
7. [5.7 Conservation of Energy](#)
8. [5.8 Power](#)
9. [5.9 Work, Energy, and Power in Humans](#)
10. [5.10 World Energy Use](#)

5.1 Introduction to Work, and Energy

class="introduction"

How many
forms of
energy can
you identify
in this
photograph
of a wind
farm in
Iowa?
(credit:
Jürgen from
Sandesneben
, Germany,
Wikimedia
Commons)



Energy plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods, to the energy we use to run our cars, to the sunlight that warms us on the beach. You can also cite examples of what people call energy that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is

involved in almost all phenomena, and is one of the most important concepts of physics. What makes it even more important is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is *conserved*.

Conservation of energy (as physicists like to call the principle that energy can neither be created nor destroyed) is based on experiment. Even as scientists discovered new forms of energy, conservation of energy has always been found to apply. Perhaps the most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy (his famous equation $E = mc^2$).

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world's energy use patterns at the end of this chapter.

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define **energy** as the ability to do work, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.

5.2 Work: The Scientific Definition

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.

What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done, a force must be exerted and there must be motion or displacement in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

Equation:

where W is work, d is the displacement of the system, and θ is the angle between the force vector F and the displacement vector d , as in [\[link\]](#). We can also write this as

Equation:

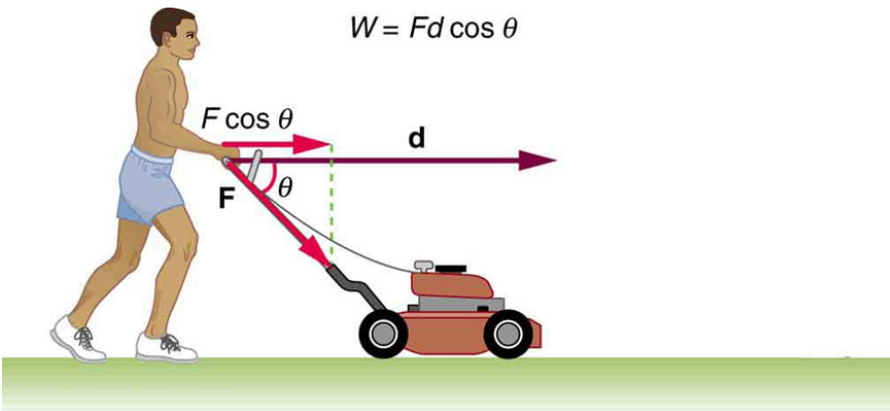
To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

Note:**What is Work?**

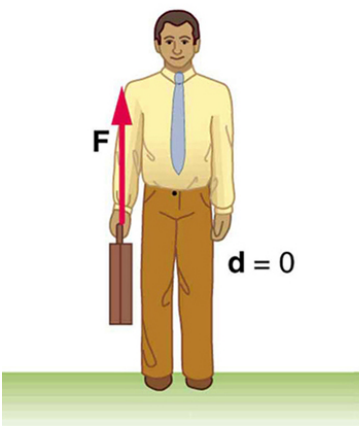
The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

Equation:

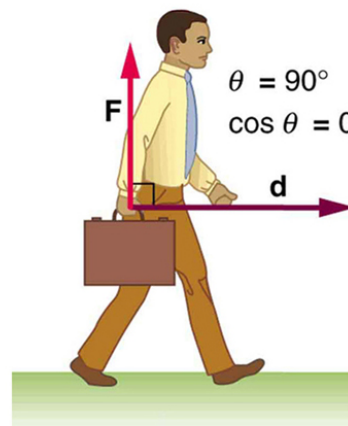
where W is work, F is the magnitude of the force on the system, d is the magnitude of the displacement of the system, and θ is the angle between the force vector \vec{F} and the displacement vector \vec{d} .



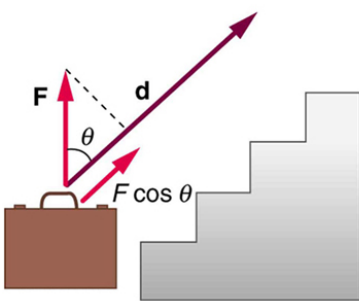
(a)



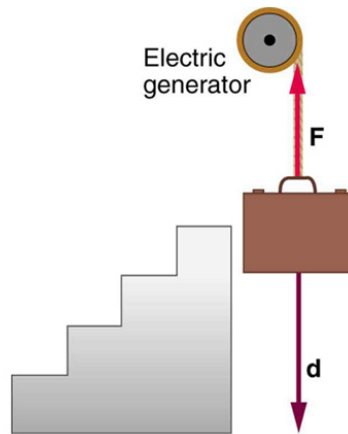
(b)



(c)



(d)



(e)

Examples of work. (a) The work done by the force on this lawn mower is . Note that is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no

motion. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work is done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because and are in opposite directions.

To examine what the definition of work means, let us consider the other situations shown in [\[link\]](#). The person holding the briefcase in [\[link\]\(b\)](#) does no work, for example. Here , so . Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the “briefcase-Earth system”—see [Gravitational Potential Energy](#) for more details). There must be motion for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in [\[link\]\(c\)](#) does no work on it, because the force is perpendicular to the motion. That is, $\theta = 90^\circ$, and so $\cos 90^\circ = 0$.

In contrast, when a force exerted on the system has a component in the direction of motion, such as in [\[link\]\(d\)](#), work is done—energy is transferred to the briefcase. Finally, in [\[link\]\(e\)](#), energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase’s weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter, with the force from the generator upward

on the briefcase, and the displacement downward. This makes $\theta = 90^\circ$, and $\cos 90^\circ = 0$; therefore, W is negative.

Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in **newton-meters**. A newton-meter is given the special name **joule** (J), and $1 \text{ J} = 1 \text{ N} \cdot \text{m}$. One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

Example:

Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in [\[link\]](#)(a) if he exerts a constant force of 200 N at an angle 30° below the horizontal and pushes the mower 10 m on level ground? Convert the amount of work from joules to kilocalories and compare it with this person's average daily intake of 2500 kcal (about 10^7 J) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by 1°C , and is equivalent to 4.18 J , while one *food calorie* (1 kcal) is equivalent to 4180 J .

Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation $W = F \cos \theta \Delta x$. The force, angle, and displacement are given, so that only the work W is unknown.

Solution

The equation for the work is

Equation:

Substituting the known values gives

Equation:

o

Converting the work in joules to kilocalories yields
 . The ratio of the work done
 to the daily consumption is

Equation:**Discussion**

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we “work” all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

Section Summary

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work that a force does on an object is the product of the magnitude of the force, times the magnitude of the displacement, times the cosine of the angle between them. In symbols,

Equation:

- The SI unit for work and energy is the joule (J), where
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.

- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

Conceptual Questions

Exercise:

Problem:

Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

Problems & Exercises

Exercise:

Problem:

How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.

Solution:

Equation:

Exercise:

Problem:

A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.

Exercise:

Problem:

(a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. (b) What is the work done on the lift by the gravitational force in this process? (c) What is the total work done on the lift?

Solution:

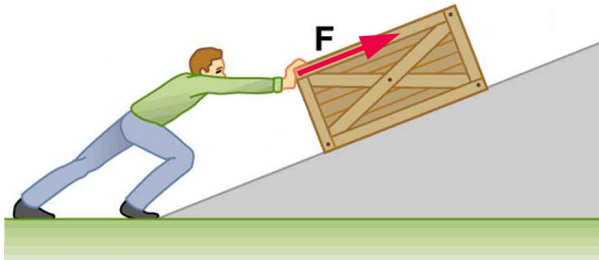
(a)

(b)

(c) The net force is zero.

Exercise:**Problem:**

Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 30° with the horizontal. (See [\[link\]](#).) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate *and* on his body to get up the ramp.



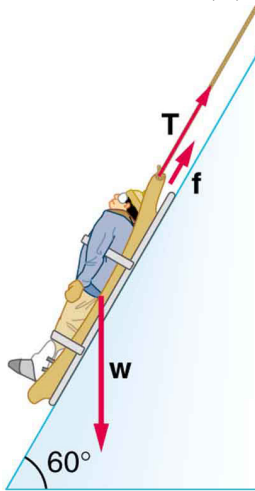
A man pushes a crate up a
ramp.

Solution:**Equation:**

Exercise:

Problem:

Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60° slope at constant speed, as shown in [\[link\]](#). The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?



A rescue sled and victim are lowered down a steep slope.

Glossary

energy

the ability to do work

work

the transfer of energy by a force that causes an object to be displaced;
the product of the component of the force in the direction of the
displacement and the magnitude of the displacement

joule

SI unit of work and energy, equal to one newton-meter

5.3 Kinetic Energy and the Work-Energy Theorem

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if the lawn mower in [\[link\]](#) (a) is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in [\[link\]](#) (d) is stored in the briefcase-Earth system and can be recovered at any time, as shown in [\[link\]](#) (e). In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

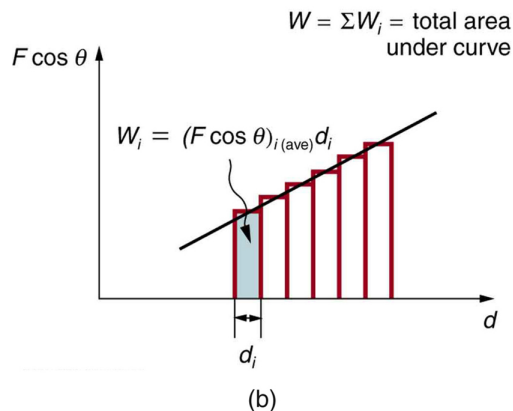
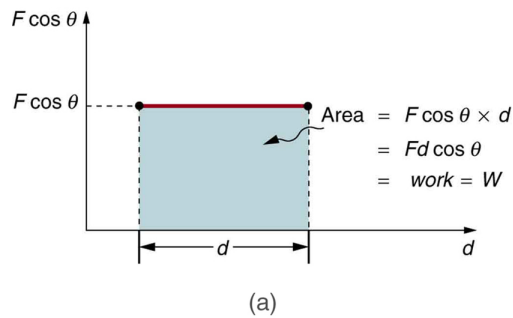
In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

Net Work and the Work-Energy Theorem

We know from the study of Newton's laws in [Dynamics: Force and Newton's Laws of Motion](#) that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work done by all external forces—that is, **net work** is the work done by the net external force \mathbf{F}_{net} . In equation form, this is $W_{\text{net}} = F_{\text{net}}d \cos \theta$ where θ is the angle between the force vector and the displacement vector.

[\[link\]](#)(a) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an $F \cos \theta$ vs. d graph. In this case, $F \cos \theta$ is constant. You can see that the area under the graph is $Fd \cos \theta$, or the work done. [\[link\]](#)(b) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force $(F \cos \theta)_{i(\text{ave})}$. The work done is $(F \cos \theta)_{i(\text{ave})}d_i$ for each strip, and the total work done is the sum of the W_i . Thus the total work done is the total area under the curve, a useful property to which we shall refer later.

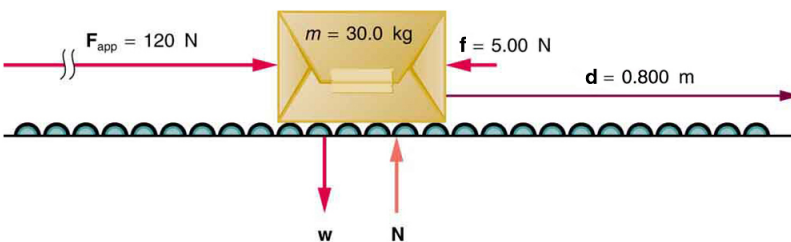


(a) A graph of $F \cos \theta$ vs. d , when $F \cos \theta$ is

constant. The area under the curve represents the work done by the force.

(b) A graph of $F \cos \theta$ vs. d in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done.

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in [\[link\]](#).



A package on a roller belt is pushed horizontally through a distance d .

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force F_{app} and the horizontal friction force f . Thus, as expected, the net force is

parallel to the displacement, so that $\theta = 0^\circ$ and $\cos \theta = 1$, and the net work is given by

Equation:

$$W_{\text{net}} = F_{\text{net}}d.$$

The effect of the net force \mathbf{F}_{net} is to accelerate the package from v_0 to v . The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See [\[link\]](#).) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting $F_{\text{net}} = ma$ from Newton's second law gives

Equation:

$$W_{\text{net}} = mad.$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $d = x - x_0$ and use the equation studied in [Motion Equations for Constant Acceleration in One Dimension](#) for the change in speed over a distance d if the acceleration has the constant value a ; namely, $v^2 = v_0^2 + 2ad$ (note that a appears in the expression for the net work). Solving for acceleration gives $a = \frac{v^2 - v_0^2}{2d}$. When a is substituted into the preceding expression for W_{net} , we obtain

Equation:

$$W_{\text{net}} = m \left(\frac{v^2 - v_0^2}{2d} \right) d.$$

The d cancels, and we rearrange this to obtain

Equation:

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

This expression is called the **work-energy theorem**, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$. This quantity is our first example of a form of energy.

Note:

The Work-Energy Theorem

The net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.

Equation:

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

The quantity $\frac{1}{2}mv^2$ in the work-energy theorem is defined to be the translational **kinetic energy** (KE) of a mass m moving at a speed v . (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,

Equation:

$$\text{KE} = \frac{1}{2}mv^2,$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in [\[link\]](#), up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50

km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

Example:

Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in [\[link\]](#) is moving at 0.500 m/s. What is its kinetic energy?

Strategy

Because the mass m and speed v are given, the kinetic energy can be calculated from its definition as given in the equation $\text{KE} = \frac{1}{2}mv^2$.

Solution

The kinetic energy is given by

Equation:

$$\text{KE} = \frac{1}{2}mv^2.$$

Entering known values gives

Equation:

$$\text{KE} = 0.5(30.0 \text{ kg})(0.500 \text{ m/s})^2,$$

which yields

Equation:

$$\text{KE} = 3.75 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 3.75 \text{ J}.$$

Discussion

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

Example:**Determining the Work to Accelerate a Package**

Suppose that you push on the 30.0-kg package in [\[link\]](#) with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.

(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

Strategy and Concept for (a)

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See [\[link\]](#).) As expected, the net work is the net force times distance.

Solution for (a)

The net force is the push force minus friction, or

$F_{\text{net}} = 120 \text{ N} - 5.00 \text{ N} = 115 \text{ N}$. Thus the net work is

Equation:

$$\begin{aligned} W_{\text{net}} &= F_{\text{net}}d = (115 \text{ N})(0.800 \text{ m}) \\ &= 92.0 \text{ N} \cdot \text{m} = 92.0 \text{ J}. \end{aligned}$$

Discussion for (a)

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

Strategy and Concept for (b)

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

Solution for (b)

The applied force does work.

Equation:

$$\begin{aligned}
 W_{\text{app}} &= F_{\text{app}} d \cos(0^\circ) = F_{\text{app}} d \\
 &= (120 \text{ N})(0.800 \text{ m}) \\
 &= 96.0 \text{ J}
 \end{aligned}$$

The friction force and displacement are in opposite directions, so that $\theta = 180^\circ$, and the work done by friction is

Equation:

$$\begin{aligned}
 W_{\text{fr}} &= F_{\text{fr}} d \cos(180^\circ) = -F_{\text{fr}} d \\
 &= -(5.00 \text{ N})(0.800 \text{ m}) \\
 &= -4.00 \text{ J.}
 \end{aligned}$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

Equation:

$$\begin{aligned}
 W_{\text{gr}} &= 0, \\
 W_{\text{N}} &= 0, \\
 W_{\text{app}} &= 96.0 \text{ J}, \\
 W_{\text{fr}} &= -4.00 \text{ J.}
 \end{aligned}$$

The total work done as the sum of the work done by each force is then seen to be

Equation:

$$W_{\text{total}} = W_{\text{gr}} + W_{\text{N}} + W_{\text{app}} + W_{\text{fr}} = 92.0 \text{ J.}$$

Discussion for (b)

The calculated total work W_{total} as the sum of the work by each force agrees, as expected, with the work W_{net} done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

Example:

Determining Speed from Work and Energy

Find the speed of the package in [\[link\]](#) at the end of the push, using work and energy concepts.

Strategy

Here the work-energy theorem can be used, because we have just calculated the net work, W_{net} , and the initial kinetic energy, $\frac{1}{2}mv_0^2$. These calculations allow us to find the final kinetic energy, $\frac{1}{2}mv^2$, and thus the final speed v .

Solution

The work-energy theorem in equation form is

Equation:

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

Solving for $\frac{1}{2}mv^2$ gives

Equation:

$$\frac{1}{2}mv^2 = W_{\text{net}} + \frac{1}{2}mv_0^2.$$

Thus,

Equation:

$$\frac{1}{2}mv^2 = 92.0 \text{ J} + 3.75 \text{ J} = 95.75 \text{ J}.$$

Solving for the final speed as requested and entering known values gives

Equation:

$$\begin{aligned} v &= \sqrt{\frac{2(95.75 \text{ J})}{m}} = \sqrt{\frac{191.5 \text{ kg}\cdot\text{m}^2/\text{s}^2}{30.0 \text{ kg}}} \\ &= 2.53 \text{ m/s}. \end{aligned}$$

Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work

done on the package. This means that the work indeed adds to the energy of the package.

Example:

Work and Energy Can Reveal Distance, Too

How far does the package in [\[link\]](#) coast after the push, assuming friction remains constant? Use work and energy considerations.

Strategy

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package's kinetic energy. The work done by friction is the force of friction times the distance traveled times the cosine of the angle between the friction force and displacement; hence, this gives us a way of finding the distance traveled after the person stops pushing.

Solution

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement, so $\theta = 180^\circ$. To reduce the kinetic energy of the package to zero, the work W_{fr} by friction must be minus the kinetic energy that the package started with plus what the package accumulated due to the pushing. Thus $W_{\text{fr}} = -95.75 \text{ J}$. Furthermore, $W_{\text{fr}} = f d' \cos \theta = -f d'$, where d' is the distance it takes to stop. Thus,

Equation:

$$d' = -\frac{W_{\text{fr}}}{f} = -\frac{-95.75 \text{ J}}{5.00 \text{ N}},$$

and so

Equation:

$$d' = 19.2 \text{ m}.$$

Discussion

This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the

force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

Section Summary

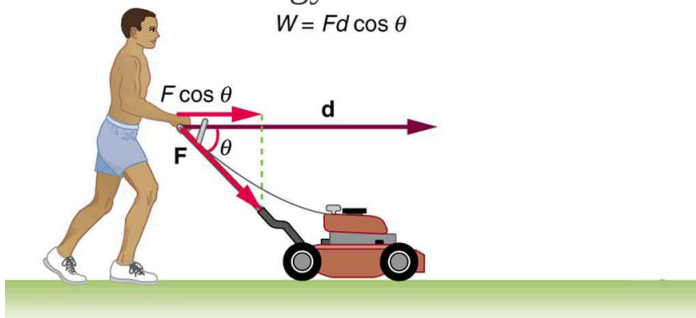
- The net work W_{net} is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is $\text{KE} = \frac{1}{2}mv^2$.
- The work-energy theorem states that the net work W_{net} on a system changes its kinetic energy, $W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$.

Conceptual Questions

Exercise:

Problem:

The person in [\[link\]](#) does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?



Exercise:**Problem:**

Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

Problems & Exercises**Exercise:****Problem:**

How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s?

Exercise:**Problem:**

(a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

Exercise:**Problem:**

A car's bumper is designed to withstand a 4.0-km/h (1.1-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.

Solution:

$$2.8 \times 10^3 \text{ N}$$

Exercise:**Problem:**

Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

Solution:

102 N

Glossary

net work

work done by the net force, or vector sum of all the forces, acting on an object

work-energy theorem

the result, based on Newton's laws, that the net work done on an object is equal to its change in kinetic energy

kinetic energy

the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ for the translational (i.e., non-rotational) motion of an object of mass m moving at speed v

5.4 Gravitational Potential Energy

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass m at height h on Earth is given by $PE_g = mgh$.
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

Work Done Against Gravity

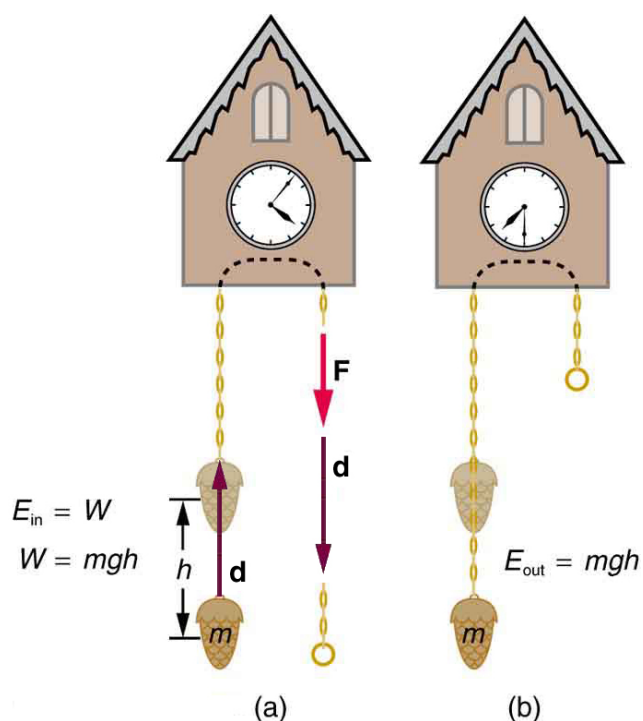
Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass m through a height h , such as in [\[link\]](#). If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight mg . The work done on the mass is then $W = Fd = mgh$. We define this to be the **gravitational potential energy** (PE_g) put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the PE_g gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word “system”? Potential energy is a property of a system rather than of a single object—due to its physical position. An object’s gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth’s surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work

equal to mgh on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of PE_g to KE without explicitly considering the intermediate step of work. (See [\[link\]](#).) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.



(a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy ΔPE_g to be
Equation:

$$\Delta PE_g = mgh,$$

where, for simplicity, we denote the change in height by h rather than the usual Δh . Note that h is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

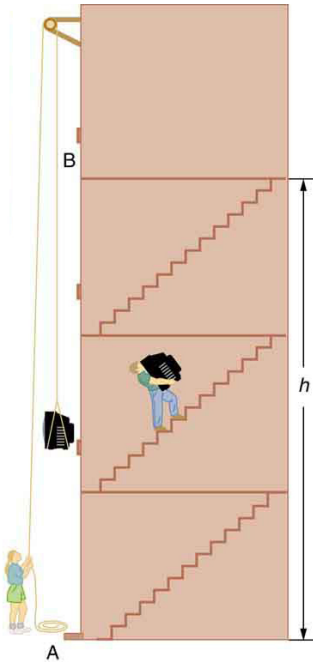
Equation:

$$\begin{aligned} mgh &= (0.500 \text{ kg}) (9.80 \text{ m/s}^2) (1.00 \text{ m}) \\ &= 4.90 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 4.90 \text{ J}. \end{aligned}$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work*.

Using Potential Energy to Simplify Calculations

The equation $\Delta \text{PE}_g = mgh$ applies for any path that has a change in height of h , not just when the mass is lifted straight up. (See [\[link\]](#).) It is much easier to calculate mgh (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position h of a mass m is accompanied by a change in gravitational potential energy mgh , and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.



The change in
gravitational
potential energy
(ΔPE_g)
between points
A and B is
independent of
the path.

$\Delta PE_g = mgh$
for any path
between the two
points. Gravity
is one of a small
class of forces
where the work
done by or
against the force
depends only on
the starting and
ending points,
not on the path
between them.

Example:**The Force to Stop Falling**

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

Strategy

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial PE_g is transformed into KE as he falls. The work done by the floor reduces this kinetic energy to zero.

Solution

The work done on the person by the floor as he stops is given by

Equation:

$$W = Fd \cos \theta = -Fd,$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions ($\cos \theta = \cos 180^\circ = -1$). The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height h :

Equation:

$$KE = -\Delta PE_g = -mgh,$$

The distance d that the person's knees bend is much smaller than the height h of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.

The work W done by the floor on the person stops the person and brings the person's kinetic energy to zero:

Equation:

$$W = -KE = mgh.$$

Combining this equation with the expression for W gives

Equation:

$$-Fd = mgh.$$

Recalling that h is negative because the person fell *down*, the force on the knee joints is given by

Equation:

$$F = -\frac{mgh}{d} = -\frac{(60.0 \text{ kg})(9.80 \text{ m/s}^2)(-3.00 \text{ m})}{5.00 \times 10^{-3} \text{ m}} = 3.53 \times 10^5 \text{ N}.$$

Discussion

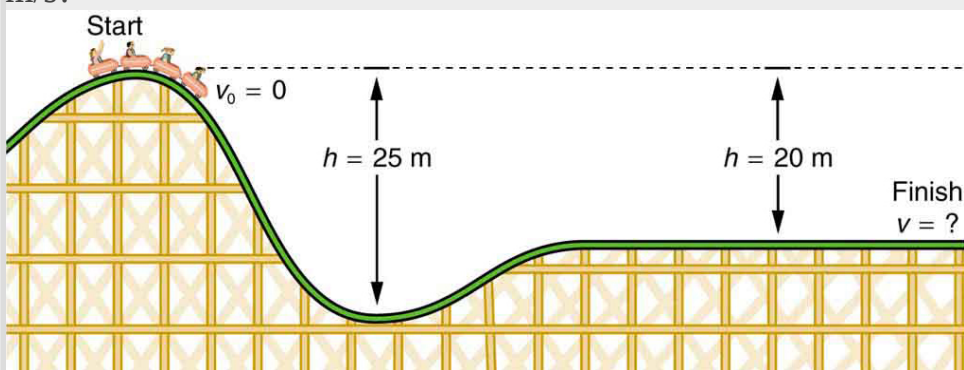
Such a large force (500 times more than the person's weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump. (See [\[link\]](#).)



The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced.
(credit: Chris Samuel, Flickr)

Example:**Finding the Speed of a Roller Coaster from its Height**

(a) What is the final speed of the roller coaster shown in [\[link\]](#) if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?



The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all ΔPE_g is converted to KE.

Strategy

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance h equals the *gain* in kinetic energy. This can be written in equation form as $-\Delta PE_g = \Delta KE$. Using the equations for PE_g and KE, we can solve for the final speed v , which is the desired quantity.

Solution for (a)

Here the initial kinetic energy is zero, so that $\Delta KE = \frac{1}{2}mv^2$. The equation for change in potential energy states that $\Delta PE_g = mgh$. Since h is negative in this case, we will rewrite this as $\Delta PE_g = -mg |h|$ to show the minus sign clearly. Thus,

Equation:

$$-\Delta PE_g = \Delta KE$$

becomes

Equation:

$$mg | h | = \frac{1}{2}mv^2.$$

Solving for v , we find that mass cancels and that

Equation:

$$v = \sqrt{2g | h |}.$$

Substituting known values,

Equation:

$$\begin{aligned} v &= \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m})} \\ &= 19.8 \text{ m/s.} \end{aligned}$$

Solution for (b)

Again $-\Delta PE_g = \Delta KE$. In this case there is initial kinetic energy, so

$\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$. Thus,

Equation:

$$mg | h | = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

Rearranging gives

Equation:

$$\frac{1}{2}mv^2 = mg | h | + \frac{1}{2}mv_0^2.$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

Equation:

$$v = \sqrt{2g | h | + v_0^2}.$$

This equation is very similar to the kinematics equation $v = \sqrt{v_0^2 + 2ad}$, but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

Equation:

$$\begin{aligned} v &= \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m}) + (5.00 \text{ m/s})^2} \\ &= 20.4 \text{ m/s.} \end{aligned}$$

Discussion and Implications

First, note that mass cancels. This is quite consistent with observations made in [Falling Objects](#) that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of h at the point of interest.

We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

Section Summary

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy, ΔPE_g , is $\Delta PE_g = mgh$, with h being the increase in height and g the acceleration due to gravity.
- The gravitational potential energy of an object near Earth's surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy, ΔPE_g , have physical significance.

- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that $\Delta KE = -\Delta PE_g$.

Conceptual Questions

Exercise:

Problem:

Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

Problems & Exercises

Exercise:

Problem:

A hydroelectric power facility (see [\[link\]](#)) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume 50.0 km^3 (mass $= 5.00 \times 10^{13} \text{ kg}$), given that the lake has an average height of 40.0 m above the generators?



Hydroelectric facility (credit: Denis Belevich, Wikimedia Commons)

Solution:

(a) $1.96 \times 10^{16} \text{ J}$

Exercise:**Problem:**

A 100-g toy car is propelled by a compressed spring that starts it moving. The car follows the curved track in [\[link\]](#). Show that the final speed of the toy car is 0.687 m/s if its initial speed is 2.00 m/s and it coasts up the frictionless slope, gaining 0.180 m in altitude.



A toy car moves up a sloped track.
(credit: Leszek Leszczynski, Flickr)

Solution:**Equation:**

$$v_f = \sqrt{2gh + v_0^2} = \sqrt{2(9.80 \text{ m/s}^2)(-0.180 \text{ m}) + (2.00 \text{ m/s})^2} = 0.687 \text{ m/s}$$

Exercise:

Problem:

In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who skies 70.0 m along a 30° slope neglecting friction: (a) Starting from rest. (b) Starting with an initial speed of 2.50 m/s. (c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

Glossary

gravitational potential energy

the energy an object has due to its position in a gravitational field

5.5 Conservative Forces and Potential Energy

- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke's law applies.
- Use the work-energy theorem to show how having only conservative forces implies conservation of mechanical energy.

Potential Energy and Conservative Forces

Work is done by a force, and some forces, such as weight, have special characteristics. A **conservative force** is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a **potential energy** for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

Note:

Potential Energy and Conservative Forces

Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.

A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

We can define a potential energy for any conservative force. The work done against a conservative force to reach a final configuration

depends on the configuration, not the path followed, and is the potential energy added.

Potential Energy of a Spring

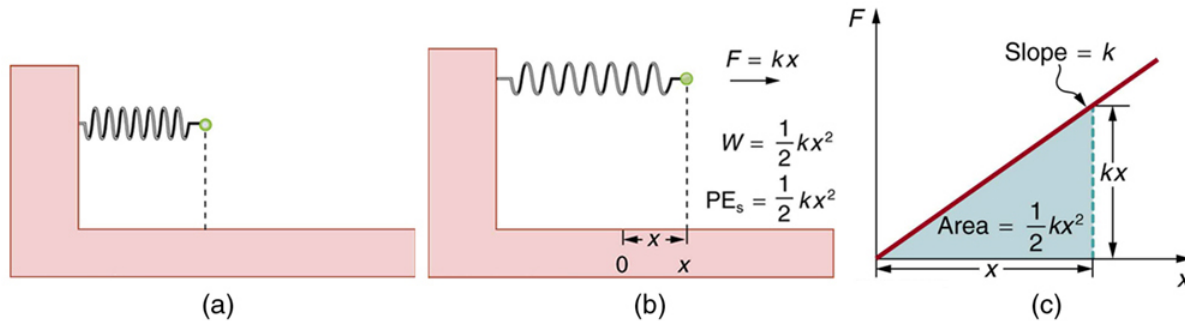
First, let us obtain an expression for the potential energy stored in a spring (). We calculate the work done to stretch or compress a spring that obeys Hooke's law. (Hooke's law was examined in [Elasticity: Stress and Strain](#), and states that the magnitude of force on the spring and the resulting deformation are proportional, .) (See [\[link\]](#).) For our spring, we will replace (the amount of deformation produced by a force) by the distance that the spring is stretched or compressed along its length. So the force needed to stretch the spring has magnitude , where is the spring's force constant. The force increases linearly from 0 at the start to in the fully stretched position. The average force is . Thus the work done in stretching or compressing the spring is

— — . Alternatively, we noted in [Kinetic Energy and the Work-Energy Theorem](#) that the area under a graph of vs. is the work done by the force. In [\[link\]](#)(c) we see that this area is also — . We therefore define the **potential energy of a spring**, , to be

Equation:

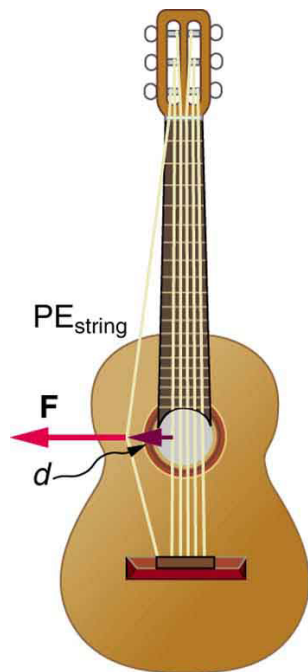
—

where is the spring's force constant and is the displacement from its undeformed position. The potential energy represents the work done on the spring and the energy stored in it as a result of stretching or compressing it a distance . The potential energy of the spring does not depend on the path taken; it depends only on the stretch or squeeze in the final configuration.



- (a) An undeformed spring has no stored in it. (b) The force needed to stretch (or compress) the spring a distance has a magnitude , and the work done to stretch (or compress) it is . Because the force is conservative, this work is stored as potential energy in the spring, and it can be fully recovered. (c) A graph of vs. has a slope of , and the area under the graph is . Thus the work done or potential energy stored is .

The equation — has general validity beyond the special case for which it was derived. Potential energy can be stored in any elastic medium by deforming it. Indeed, the general definition of **potential energy** is energy due to position, shape, or configuration. For shape or position deformations, stored energy is — , where is the force constant of the particular system and is its deformation. Another example is seen in [\[link\]](#) for a guitar string.



Work is done
to deform the
guitar string,
giving it
potential
energy.

When
released, the
potential
energy is
converted to
kinetic
energy and
back to
potential as
the string
oscillates
back and
forth. A very
small
fraction is
dissipated as

sound
energy,
slowly
removing
energy from
the string.

Conservation of Mechanical Energy

Let us now consider what form the work-energy theorem takes when only conservative forces are involved. This will lead us to the conservation of energy principle. The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

Equation:

$$W_{\text{net}} = \Delta K$$

If only conservative forces act, then

Equation:

$$W_{\text{c}} = \Delta K$$

where W_{c} is the total work done by all conservative forces. Thus,

Equation:

$$W_{\text{c}} = \Delta K$$

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is,

Therefore,

Equation:

$$W_{\text{c}} = \Delta K$$

or

Equation:

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

Equation:

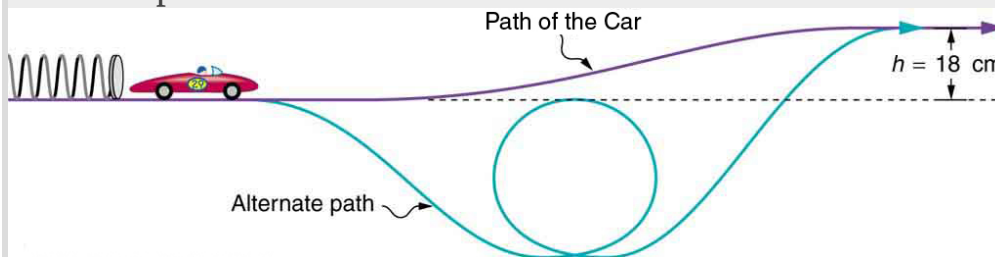
where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy** principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its **mechanical energy**, E_{mech} . In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between E_{mech} and the various types of E_{mech} , with the total energy remaining constant.

Example:

Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in [\[link\]](#). The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car

is going before it starts up the slope and (b) how fast it is going at the top of the slope.



A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown.

Strategy

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

Equation:

or

Equation:

— — — —

where h is the height (vertical position) and x is the compression of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

Solution for (a)

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both U_i and U_f are zero. Furthermore, the initial speed v_i is zero and the final compression of the spring x_f is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

Equation:

$$\frac{1}{2}mv_f^2 = \frac{1}{2}kx_i^2$$

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the absence of friction. Solving for the final speed and entering known values yields

Equation:

$$v_f = \sqrt{\frac{kx_i^2}{m}}$$

Solution for (b)

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

Equation:

$$U_i + K_i = U_f + K_f$$

This form of the equation means that the spring's initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for v_f and substituting known values gives

Equation:

Discussion

Another way to solve this problem is to realize that the car's kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Section Summary

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy U for any conservative force, just as we defined U_g for the gravitational force.
- The potential energy of a spring is $U_s = \frac{1}{2}kx^2$, where k is the spring's force constant and x is the displacement from its undeformed position.
- Mechanical energy is defined to be $E = K + U$ for a conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is constant. In equation form,

Equation:

where i and f denote initial and final values. This is known as the conservation of mechanical energy.

Conceptual Questions

Exercise:

Problem: What is a conservative force?

Exercise:

Problem:

The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after his feet leave it.

Problems & Exercises

Exercise:

Problem:

A subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the force constant of the spring?

Solution:

Equation:

Exercise:

Problem:

A pogo stick has a spring with a force constant of _____, which can be compressed 12.0 cm. To what maximum height can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40.0 kg? Explicitly show how you follow the steps in the [Problem-Solving Strategies for Energy](#).

Glossary

conservative force

a force that does the same work for any given initial and final configuration, regardless of the path followed

potential energy

energy due to position, shape, or configuration

potential energy of a spring

the stored energy of a spring as a function of its displacement; when Hooke's law applies, it is given by the expression $\frac{1}{2}kx^2$ where x is the distance the spring is compressed or extended and k is the spring constant

conservation of mechanical energy

the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system

mechanical energy

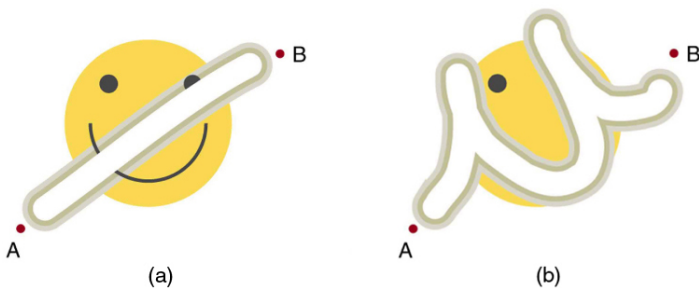
the sum of kinetic energy and potential energy

5.6 Nonconservative Forces

- Define nonconservative forces and explain how they affect mechanical energy.
- Show how the principle of conservation of energy can be applied by treating the conservative forces in terms of their potential energies and any nonconservative forces in terms of the work they do.

Nonconservative Forces and Friction

Forces are either conservative or nonconservative. Conservative forces were discussed in [Conservative Forces and Potential Energy](#). A **nonconservative force** is one for which work depends on the path taken. Friction is a good example of a nonconservative force. As illustrated in [\[link\]](#), work done against friction depends on the length of the path between the starting and ending points. Because of this dependence on path, there is no potential energy associated with nonconservative forces. An important characteristic is that the work done by a nonconservative force *adds or removes mechanical energy from a system*. **Friction**, for example, creates **thermal energy** that dissipates, removing energy from the system. Furthermore, even if the thermal energy is retained or captured, it cannot be fully converted back to work, so it is lost or not recoverable in that sense as well.

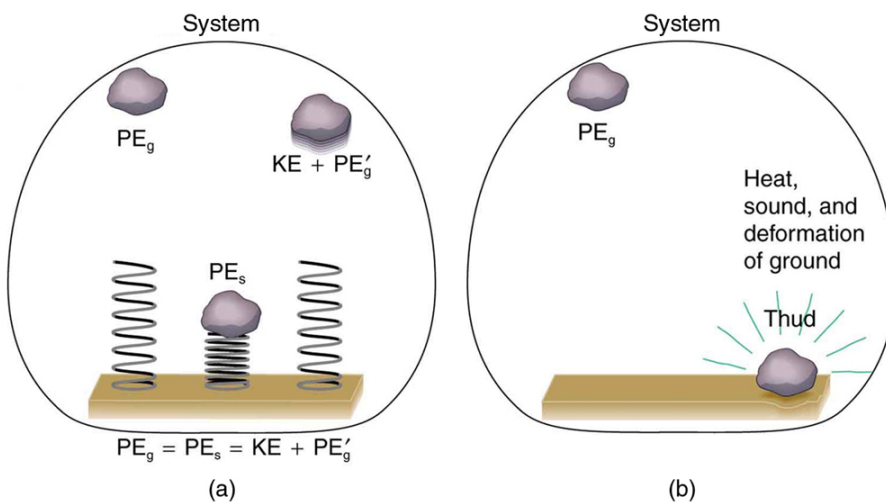


The amount of the happy face erased depends on the path taken by the eraser between points A and B, as does the work done against friction. Less work is done and less of the face

is erased for the path in (a) than for the path in (b). The force here is friction, and most of the work goes into thermal energy that subsequently leaves the system (the happy face plus the eraser). The energy expended cannot be fully recovered.

How Nonconservative Forces Affect Mechanical Energy

Mechanical energy may not be conserved when nonconservative forces act. For example, when a car is brought to a stop by friction on level ground, it loses kinetic energy, which is dissipated as thermal energy, reducing its mechanical energy. [\[link\]](#) compares the effects of conservative and nonconservative forces. We often choose to understand simpler systems such as that described in [\[link\]](#)(a) first before studying more complicated systems as in [\[link\]](#)(b).



Comparison of the effects of conservative and nonconservative forces on the mechanical energy of a system. (a) A system with only conservative

forces. When a rock is dropped onto a spring, its mechanical energy remains constant (neglecting air resistance) because the force in the spring is conservative. The spring can propel the rock back to its original height, where it once again has only potential energy due to gravity. (b) A system with nonconservative forces. When the same rock is dropped onto the ground, it is stopped by nonconservative forces that dissipate its mechanical energy as thermal energy, sound, and surface distortion. The rock has lost mechanical energy.

How the Work-Energy Theorem Applies

Now let us consider what form the work-energy theorem takes when both conservative and nonconservative forces act. We will see that the work done by nonconservative forces equals the change in the mechanical energy of a system. As noted in [Kinetic Energy and the Work-Energy Theorem](#), the work-energy theorem states that the net work on a system equals the change in its kinetic energy, or $W_{\text{net}} = \Delta K$. The net work is the sum of the work by nonconservative forces plus the work by conservative forces. That is,

Equation:

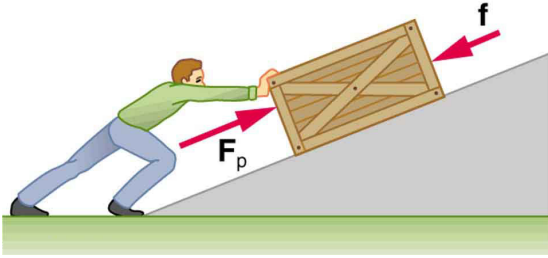
$$W_{\text{nc}} + W_{\text{c}} = \Delta K$$

so that

Equation:

$$W_{\text{nc}} = \Delta K - W_{\text{c}}$$

where W_{nc} is the total work done by all nonconservative forces and W_{c} is the total work done by all conservative forces.



A person pushes a crate up a ramp, doing work on the crate. Friction and gravitational force (not shown) also do work on the crate; both forces oppose the person's push. As the crate is pushed up the ramp, it gains mechanical energy, implying that the work done by the person is greater than the work done by friction.

Consider [\[link\]](#), in which a person pushes a crate up a ramp and is opposed by friction. As in the previous section, we note that work done by a conservative force comes from a loss of gravitational potential energy, so that $W_{\text{grav}} = -\Delta U$. Substituting this equation into the previous one and solving for W_{nc} gives

Equation:

$$W_{\text{nc}} = \Delta E_{\text{mech}}$$

This equation means that the total mechanical energy changes by exactly the amount of work done by nonconservative forces. In [\[link\]](#), this is the work done by the person minus the work done by friction. So even if energy is not conserved for the system of interest (such as the crate), we know that an equal amount of work was done to cause the change in total mechanical energy.

We rearrange W_{nc} to obtain
Equation:

$$W_{nc} = \Delta E_{mech}$$

This means that the amount of work done by nonconservative forces adds to the mechanical energy of a system. If W_{nc} is positive, then mechanical energy is increased, such as when the person pushes the crate up the ramp in [\[link\]](#). If W_{nc} is negative, then mechanical energy is decreased, such as when the rock hits the ground in [\[link\]](#)(b). If W_{nc} is zero, then mechanical energy is conserved, and nonconservative forces are balanced. For example, when you push a lawn mower at constant speed on level ground, your work done is removed by the work of friction, and the mower has a constant energy.

Applying Energy Conservation with Nonconservative Forces

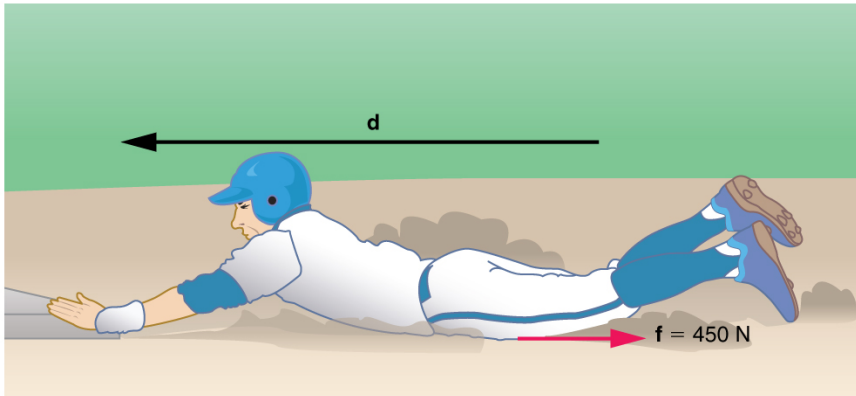
When no change in potential energy occurs, applying $W_{nc} = \Delta E_{mech}$ amounts to applying the work-energy theorem by setting the change in kinetic energy to be equal to the net work done on the system, which in the most general case includes both conservative and nonconservative forces. But when seeking instead to find a change in total mechanical energy in situations that involve changes in both potential and kinetic energy, the previous equation $W_{nc} = \Delta E_{mech}$ says that you can start by finding the change in mechanical energy that would have resulted from just the conservative forces, including the potential energy changes, and add to it the work done, with the proper sign, by any nonconservative forces involved.

Example:

Calculating Distance Traveled: How Far a Baseball Player Slides

Consider the situation shown in [\[link\]](#), where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance

the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a constant 450 N.



The baseball player slides to a stop in a distance d . In the process, friction removes the player's kinetic energy by doing an amount of work equal to the initial kinetic energy.

Strategy

Friction stops the player by converting his kinetic energy into other forms, including thermal energy. In terms of the work-energy theorem, the work done by friction, which is negative, is added to the initial kinetic energy to reduce it to zero. The work done by friction is negative, because \vec{f} is in the opposite direction of the motion (that is, $\theta = 180^\circ$, and so $\cos \theta = -1$). Thus $W = -fd$. The equation simplifies to

Equation:

$$-\frac{1}{2}mv^2 = -fd$$

or

Equation:

$$\frac{1}{2}mv^2 = fd$$

This equation can now be solved for the distance d .

Solution

Solving the previous equation for d and substituting known values yields
Equation:

$$d = \frac{mv}{f}$$

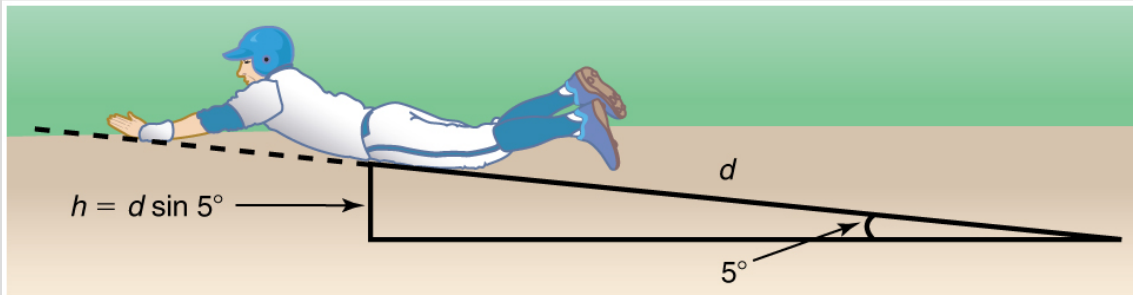
Discussion

The most important point of this example is that the amount of nonconservative work equals the change in mechanical energy. For example, you must work harder to stop a truck, with its large mechanical energy, than to stop a mosquito.

Example:

Calculating Distance Traveled: Sliding Up an Incline

Suppose that the player from [\[link\]](#) is running up a hill having a 5° incline upward with a surface similar to that in the baseball stadium. The player slides with the same initial speed. Determine how far he slides.



The same baseball player slides to a stop on a 5° slope.

Strategy

In this case, the work done by the nonconservative friction force on the player reduces the mechanical energy he has from his kinetic energy at zero height, to the final mechanical energy he has by moving through

distance d to reach height h along the hill, with $h = d \sin \theta$. This is expressed by the equation

Equation:

$$W = f d$$

Solution

The work done by friction is again $W = f d$; initially the potential energy is 0 and the kinetic energy is $\frac{1}{2} m v^2$; the final energy contributions are 0 for the kinetic energy and $m g d \sin \theta$ for the potential energy.

Substituting these values gives

Equation:

$$\frac{1}{2} m v^2 = f d + m g d \sin \theta$$

Solve this for d to obtain

Equation:

$$d = \frac{\frac{1}{2} m v^2}{f + m g \sin \theta}$$

Discussion

As might have been expected, the player slides a shorter distance by sliding uphill. Note that the problem could also have been solved in terms of the forces directly and the work energy theorem, instead of using the potential energy. This method would have required combining the normal force and force of gravity vectors, which no longer cancel each other because they point in different directions, and friction, to find the net force. You could then use the net force and the net work to find the distance d that reduces the kinetic energy to zero. By applying conservation of energy and using the potential energy instead, we need only consider the gravitational potential energy, without combining and resolving force vectors. This simplifies the solution considerably.

Section Summary

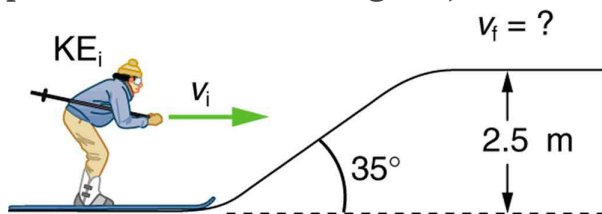
- A nonconservative force is one for which work depends on the path.
- Friction is an example of a nonconservative force that changes mechanical energy into thermal energy.
- Work W done by a nonconservative force changes the mechanical energy of a system. In equation form, $W = \Delta E_{\text{mech}}$ or, equivalently, $W = E_{\text{mech}} - E_{\text{mech},i}$.
- When both conservative and nonconservative forces act, energy conservation can be applied and used to calculate motion in terms of the known potential energies of the conservative forces and the work done by nonconservative forces, instead of finding the net work from the net force, or having to directly apply Newton's laws.

Problems & Exercises

Exercise:

Problem:

A 60.0-kg skier with an initial speed of 12.0 m/s coasts up a 2.50-m-high rise as shown in [\[link\]](#). Find her final speed at the top, given that the coefficient of friction between her skis and the snow is 0.0800. (Hint: Find the distance traveled up the incline assuming a straight-line path as shown in the figure.)



The skier's initial kinetic energy is partially used in coasting to the top of a rise.

Solution:

9.46 m/s

Exercise:**Problem:**

(a) How high a hill can a car coast up (engine disengaged) if work done by friction is negligible and its initial speed is 110 km/h? (b) If, in actuality, a 750-kg car with an initial speed of 110 km/h is observed to coast up a hill to a height 22.0 m above its starting point, how much thermal energy was generated by friction? (c) What is the average force of friction if the hill has a slope $^{\circ}$ above the horizontal?

Glossary

nonconservative force

a force whose work depends on the path followed between the given initial and final configurations

friction

the force between surfaces that opposes one sliding on the other; friction changes mechanical energy into thermal energy

5.7 Conservation of Energy

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

Law of Conservation of Energy

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The **law of conservation of energy** can be stated as follows:

Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy—mechanical energy and energy transferred via work done by nonconservative forces. But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all of these before we can write an equation for the above general statement of the conservation of energy.

Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called **other energy** (). Then we can state the conservation of energy in equation form as

Equation:

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is , work done by a conservative force is represented by , work done by nonconservative forces is , and

all other energies are included as _____. This equation applies to all previous examples; in those situations _____ was constant, and so it subtracted out and was not directly considered.

Note:

Making Connections: Usefulness of the Energy Conservation Principle

The fact that energy is conserved and has many forms makes it very important. You will find that energy is discussed in many contexts, because it is involved in all processes. It will also become apparent that many situations are best understood in terms of energy and that problems are often most easily conceptualized and solved by considering energy.

When does _____ play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of _____).

Some of the Many Forms of Energy

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons.

Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

[\[link\]](#) gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

Note:

Problem-Solving Strategies for Energy

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier—involving identifying physical principles, knowns, and unknowns, checking units, and so on—continue to be relevant here.

Step 1. Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

Step 2. Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

Step 3. If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is

Equation:

Step 4. If you know the potential energy for only some of the forces, possibly because some of them are nonconservative and do not have a potential energy, or if there are other energies that are not easily treated in terms of force and work, then the conservation of energy law in its most general form must be used.

Equation:

In most problems, one or more of the terms is zero, simplifying its solution. Do not calculate $W_{\text{conservative}}$, the work done by conservative forces; it is already incorporated in the U terms.

Step 5. You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose U at either the initial or final point, so that $W_{\text{conservative}}$ is zero there. Then solve for the unknown in the customary manner.

Step 6. *Check the answer to see if it is reasonable.* Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see [\[link\]](#)) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.



Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Object/phenomenon	Energy in joules
Big Bang	
Energy released in a supernova	
Fusion of all the hydrogen in Earth's oceans	
Annual world energy use	

Object/phenomenon	Energy in joules
Large fusion bomb (9 megaton)	
1 kg hydrogen (fusion to helium)	
1 kg uranium (nuclear fission)	
Hiroshima-size fission bomb (10 kiloton)	
90,000-ton aircraft carrier at 30 knots	
1 barrel crude oil	
1 ton TNT	
1 gallon of gasoline	
Daily home electricity use (developed countries)	
Daily adult food intake (recommended)	

Object/phenomenon	Energy in joules
1000-kg car at 90 km/h	
1 g fat (9.3 kcal)	
ATP hydrolysis reaction	
1 g carbohydrate (4.1 kcal)	
1 g protein (4.1 kcal)	
Tennis ball at 100 km/h	
Mosquito	
Single electron in a TV tube beam	
Energy to break one DNA strand	

Energy of Various Objects and Phenomena

Efficiency

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The **efficiency** of an energy conversion process is defined as

Equation:

[\[link\]](#) lists some efficiencies of mechanical devices and human activities. In a coal-fired power plant, for example, about 40% of the chemical energy in the coal becomes useful electrical energy. The other 60% transforms into other (perhaps less useful) energy forms, such as thermal energy, which is then released to the environment through combustion gases and cooling towers.

Activity/device	Efficiency (%) [footnote] Representative values
Cycling and climbing	20
Swimming, surface	2
Swimming, submerged	4
Shoveling	3
Weightlifting	9
Steam engine	17
Gasoline engine	30

Activity/device	Efficiency (%) ^[footnote] Representative values
Diesel engine	35
Nuclear power plant	35
Coal power plant	42
Electric motor	98
Compact fluorescent light	20
Gas heater (residential)	90
Solar cell	10

Efficiency of the Human Body and Mechanical Devices

Section Summary

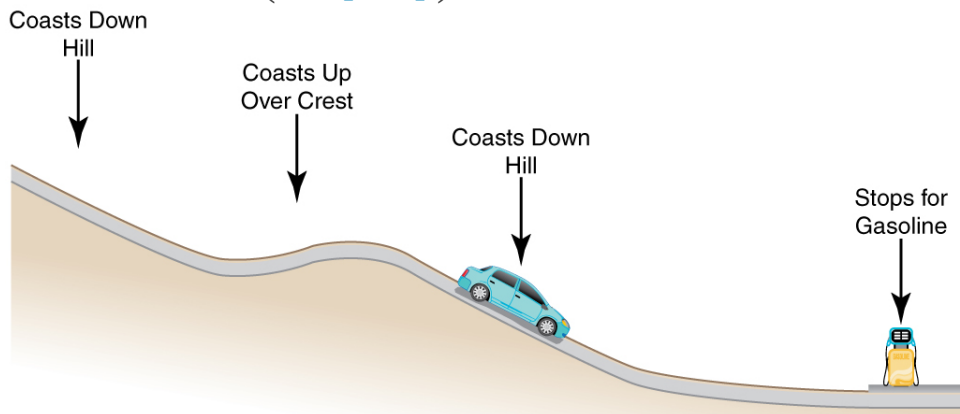
- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- When all forms of energy are considered, conservation of energy is written in equation form as $E_{\text{initial}} = E_{\text{final}}$, where E_{initial} is all **other forms of energy** besides mechanical energy.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.
- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work.
- The efficiency η of a machine or human is defined to be $\eta = \frac{W_{\text{out}}}{E_{\text{in}}}$, where W_{out} is useful work output and E_{in} is the energy consumed.

Conceptual Questions

Exercise:

Problem:

Consider the following scenario. A car for which friction is *not* negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See [\[link\]](#).)



A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station.

Exercise:

Problem:

Do devices with efficiencies of less than one violate the law of conservation of energy? Explain.

Exercise:

Problem: List the energy conversions that occur when riding a bicycle.

Problems & Exercises

Exercise:**Problem:**

Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.

Solution:

Equating _____ and _____, we obtain _____

Glossary

law of conservation of energy

the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

electrical energy

the energy carried by a flow of charge

chemical energy

the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

radiant energy

the energy carried by electromagnetic waves

nuclear energy

energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

thermal energy

the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

efficiency

a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

5.8 Power

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

What is Power?

Power—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in [\[link\]](#).



This powerful rocket on the Space Shuttle *Endeavor* did work and consumed energy at a very high rate. (credit: NASA)

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** (P) as the rate at which work is done.

Note:**Power**

Power is the rate at which work is done.

Equation:

$$P = \frac{W}{t}$$

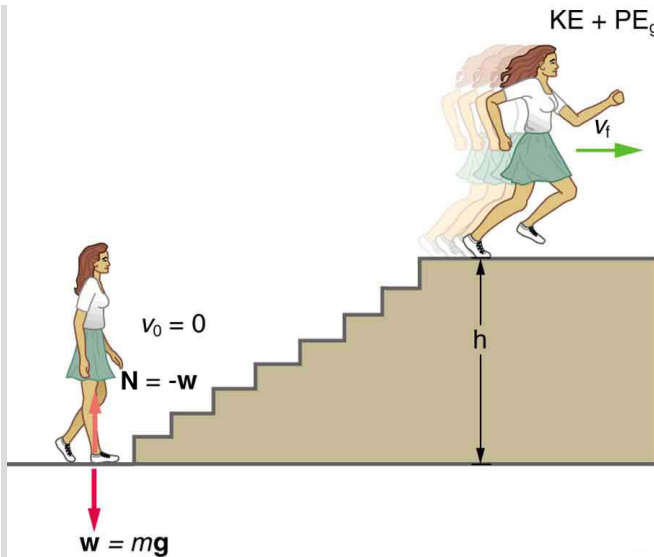
The SI unit for power is the **watt** (W), where 1 watt equals 1 joule/second ($1 \text{ W} = 1 \text{ J/s}$).

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

Calculating Power from Energy

Example:**Calculating the Power to Climb Stairs**

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See [\[link\]](#).)



When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

Strategy and Concept

The work going into mechanical energy is $W = KE + PE$. At the bottom of the stairs, we take both KE and PE_g as initially zero; thus,

$W = KE_f + PE_g = \frac{1}{2}mv_f^2 + mgh$, where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power.

Solution

Substituting the expression for W into the definition of power given in the previous equation, $P = W/t$ yields

Equation:

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv_f^2 + mgh}{t}.$$

Entering known values yields

Equation:

$$\begin{aligned}P &= \frac{0.5(60.0 \text{ kg})(2.00 \text{ m/s})^2 + (60.0 \text{ kg})(9.80 \text{ m/s}^2)(3.00 \text{ m})}{3.50 \text{ s}} \\&= \frac{120 \text{ J} + 1764 \text{ J}}{3.50 \text{ s}} \\&= 538 \text{ W}.\end{aligned}$$

Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 **horsepower** (1 hp = 746 W)! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

Note:**Making Connections: Take-Home Investigation—Measure Your Power Rating**

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

Examples of Power

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See [\[link\]](#) for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter (kW/m^2). A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is 10^6 W of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See [\[link\]](#).)



Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings.

The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)

Object or Phenomenon	Power in Watts
Supernova (at peak)	5×10^{37}
Milky Way galaxy	10^{37}
Crab Nebula pulsar	10^{28}
The Sun	4×10^{26}

Object or Phenomenon	Power in Watts
Volcanic eruption (maximum)	4×10^{15}
Lightning bolt	2×10^{12}
Nuclear power plant (total electric and heat transfer)	3×10^9
Aircraft carrier (total useful and heat transfer)	10^8
Dragster (total useful and heat transfer)	2×10^6
Car (total useful and heat transfer)	8×10^4
Football player (total useful and heat transfer)	5×10^3
Clothes dryer	4×10^3
Person at rest (all heat transfer)	100

Object or Phenomenon	Power in Watts
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Electric clock	3
Pocket calculator	10^{-3}

Power Output or Consumption

Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is $P = W/t = E/t$, where E is the energy supplied by the electricity company. So the energy consumed over a time t is

Equation:

$$E = Pt.$$

Electricity bills state the energy used in units of **kilowatt-hours** ($\text{kW} \cdot \text{h}$), which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

Example:**Calculating Energy Costs**

What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is \$0.120 per kW · h?

Strategy

Cost is based on energy consumed; thus, we must find E from $E = Pt$ and then calculate the cost. Because electrical energy is expressed in kW · h, at the start of a problem such as this it is convenient to convert the units into kW and hours.

Solution

The energy consumed in kW · h is

Equation:

$$\begin{aligned} E &= Pt = (0.200 \text{ kW})(6.00 \text{ h/d})(30.0 \text{ d}) \\ &= 36.0 \text{ kW} \cdot \text{h}, \end{aligned}$$

and the cost is simply given by

Equation:

$$\text{cost} = (36.0 \text{ kW} \cdot \text{h})(\$0.120 \text{ per kW} \cdot \text{h}) = \$4.32 \text{ per month.}$$

Discussion

The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day

usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. As we will discuss in more detail in [Thermodynamics](#), the potential for energy to produce useful work has been “degraded” in the energy transformation.

Section Summary

- Power is the rate at which work is done, or in equation form, for the average power P for work W done over a time t , $P = W/t$.
- The SI unit for power is the watt (W), where $1 \text{ W} = 1 \text{ J/s}$.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where $1 \text{ hp} = 746 \text{ W}$.

Conceptual Questions

Exercise:

Problem:

Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.

Exercise:

Problem:

Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?

Exercise:**Problem:**

A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

Problems & Exercises**Exercise:****Problem:**

What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is \$0.0900 per $\text{kW} \cdot \text{h}$?

Exercise:**Problem:**

(a) What is the average power consumption in watts of an appliance that uses $5.00 \text{ kW} \cdot \text{h}$ of energy per day? (b) How many joules of energy does this appliance consume in a year?

Exercise:**Problem:**

(a) What is the average useful power output of a person who does $6.00 \times 10^6 \text{ J}$ of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

Solution:

(a) 208 W

(b) 141 s

Exercise:**Problem:**

A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?

Exercise:**Problem:**

(a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated. (b) What does it cost, if electricity is \$0.0900 per kW · h?

Exercise:**Problem:**

(a) How long would it take a 1.50×10^5 -kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (Hint: You must find the distance the plane travels in 1200 s assuming constant acceleration.)

Exercise:

Problem:

Calculate the power output needed for a 950-kg car to climb a 2.00° slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N. Explicitly show how you follow the steps in the [Problem-Solving Strategies for Energy](#).

Solution:

Identify knowns: $m = 950$ kg, slope angle $\theta = 2.00^\circ$, $v = 30.0$ m/s, $f = 600$ N

Identify unknowns: power P of the car, force F that car applies to road

Solve for unknown:

$$P = \frac{W}{t} = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv,$$

where F is parallel to the incline and must oppose the resistive forces and the force of gravity:

$$F = f + w = 600 \text{ N} + mg \sin \theta$$

Insert this into the expression for power and solve:

$$\begin{aligned} P &= (f + mg \sin \theta)v \\ &= \left[600 \text{ N} + (950 \text{ kg})(9.80 \text{ m/s}^2) \sin 2^\circ \right] (30.0 \text{ m/s}) \\ &= 2.77 \times 10^4 \text{ W} \end{aligned}$$

About 28 kW (or about 37 hp) is reasonable for a car to climb a gentle incline.

Exercise:

Problem:

(a) Calculate the power per square meter reaching Earth's upper atmosphere from the Sun. (Take the power output of the Sun to be 4.00×10^{26} W.) (b) Part of this is absorbed and reflected by the atmosphere, so that a maximum of 1.30 kW/m^2 reaches Earth's surface. Calculate the area in km^2 of solar energy collectors needed to replace an electric power plant that generates 750 MW if the collectors convert an average of 2.00% of the maximum power into electricity. (This small conversion efficiency is due to the devices themselves, and the fact that the sun is directly overhead only briefly.) With the same assumptions, what area would be needed to meet the United States' energy needs (1.05×10^{20} J)? Australia's energy needs (5.4×10^{18} J)? China's energy needs (6.3×10^{19} J)? (These energy consumption values are from 2006.)

Glossary

power

the rate at which work is done

watt

(W) SI unit of power, with $1 \text{ W} = 1 \text{ J/s}$

horsepower

an older non-SI unit of power, with $1 \text{ hp} = 746 \text{ W}$

kilowatt-hour

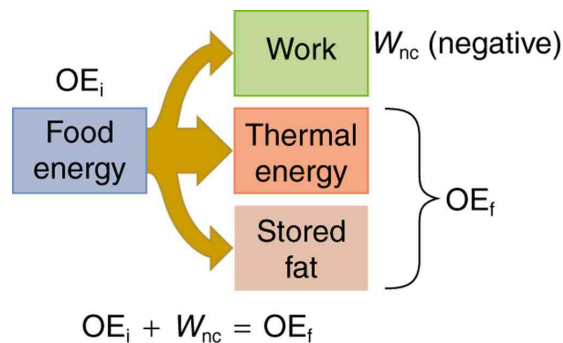
(kW · h) unit used primarily for electrical energy provided by electric utility companies

5.9 Work, Energy, and Power in Humans

- Explain the human body's consumption of energy when at rest vs. when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.

Energy Conversion in Humans

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, and/or stored as chemical energy in fatty tissue. (See [\[link\]](#).) The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.



Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity.

Power Consumed at Rest

The *rate* at which the body uses food energy to sustain life and to do different activities is called the **metabolic rate**. The total energy conversion rate of a person *at rest* is called the **basal metabolic rate** (BMR) and is divided among various systems in the body, as shown in [\[link\]](#). The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

Organ	Power consumed at rest (W)	Oxygen consumption (mL/min)	Percent of BMR
Liver & spleen	23	67	27
Brain	16	47	19
Skeletal muscle	15	45	18
Kidney	9	26	10
Heart	6	17	7
Other	16	48	19
Totals	85 W	250 mL/min	100%

Basal Metabolic Rates (BMR)

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. (See [\[link\]](#).) Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food. [\[link\]](#) shows energy and oxygen consumption rates (power expended) for a variety of activities.

Power of Doing Useful Work

Work done by a person is sometimes called **useful work**, which is *work done on the outside world*, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are nonconservative, so that they can change the mechanical energy ($KE + PE$) of the system worked upon, and this is often the goal. A baseball player throwing a ball, for example, increases both the ball's kinetic and potential energy.

If a person needs more energy than they consume, such as when doing vigorous work, the body must draw upon the chemical energy stored in fat. So exercise can be helpful in losing fat. However, the amount of exercise needed to produce a loss in fat, or to burn off extra calories consumed that day, can be large, as [\[link\]](#) illustrates.

Example:

Calculating Weight Loss from Exercising

If a person who normally requires an average of 12,000 kJ (3000 kcal) of food energy per day consumes 13,000 kJ per day, he will steadily gain weight. How much bicycling per day is required to work off this extra 1000 kJ?

Solution

[\[link\]](#) states that 400 W are used when cycling at a moderate speed. The time required to work off 1000 kJ at this rate is then

Equation:

$$\text{Time} = \frac{\text{energy}}{\left(\frac{\text{energy}}{\text{time}}\right)} = \frac{1000 \text{ kJ}}{400 \text{ W}} = 2500 \text{ s} = 42 \text{ min.}$$

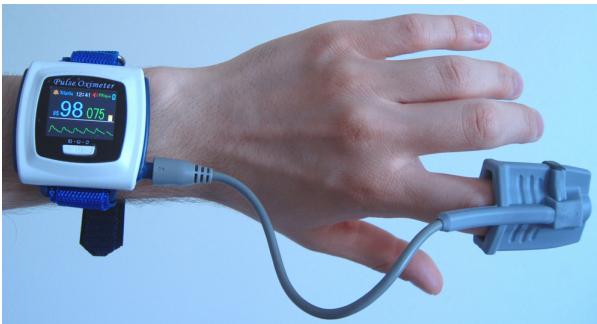
Discussion

If this person uses more energy than he or she consumes, the person's body will obtain the needed energy by metabolizing body fat. If the person uses 13,000 kJ but consumes only 12,000 kJ, then the amount of fat loss will be

Equation:

$$\text{Fat loss} = (1000 \text{ kJ}) \frac{1.0 \text{ g fat}}{39 \text{ kJ}} = 26 \text{ g,}$$

assuming the energy content of fat to be 39 kJ/g.



A pulse oxymeter is an apparatus that measures the amount of oxygen in blood.

Oxymeters can be used to determine a person's metabolic rate, which is the rate at which food energy is converted to another form. Such

measurements can indicate the level of athletic conditioning as well as certain medical problems. (credit: UusiAjaja, Wikimedia Commons)

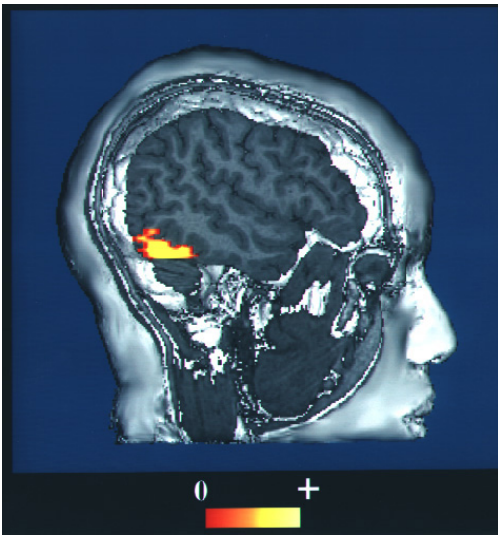
Activity	Energy consumption in watts	Oxygen consumption in liters O₂/min
Sleeping	83	0.24
Sitting at rest	120	0.34
Standing relaxed	125	0.36
Sitting in class	210	0.60
Walking (5 km/h)	280	0.80
Cycling (13–18 km/h)	400	1.14
Shivering	425	1.21
Playing tennis	440	1.26

Activity	Energy consumption in watts	Oxygen consumption in liters O₂/min
Swimming breaststroke	475	1.36
Ice skating (14.5 km/h)	545	1.56
Climbing stairs (116/min)	685	1.96
Cycling (21 km/h)	700	2.00
Running cross-country	740	2.12
Playing basketball	800	2.28
Cycling, professional racer	1855	5.30
Sprinting	2415	6.90

Energy and Oxygen Consumption Rates[\[footnote\]](#) (Power)
for an average 76-kg male

All bodily functions, from thinking to lifting weights, require energy. (See [\[link\]](#).) The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and

do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all it that a full 25% of all energy consumed by the body is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.



This fMRI scan shows an increased level of energy consumption in the vision center of the brain. Here, the patient was being asked to recognize faces.
(credit: NIH via Wikimedia Commons)

Section Summary

- The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.
- The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic rate (BMR)
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

Glossary

metabolic rate

the rate at which the body uses food energy to sustain life and to do different activities

basal metabolic rate

the total energy conversion rate of a person at rest

useful work

work done on an external system

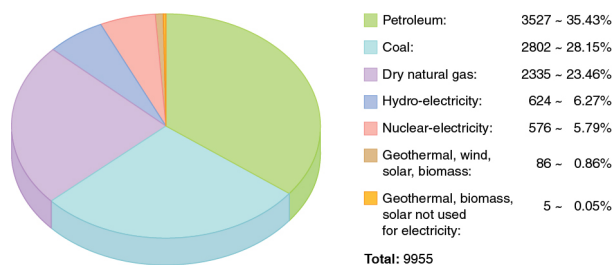
5.10 World Energy Use

- Describe the distinction between renewable and nonrenewable energy sources.
- Explain why the inevitable conversion of energy to less useful forms makes it necessary to conserve energy resources.

Energy is an important ingredient in all phases of society. We live in a very interdependent world, and access to adequate and reliable energy resources is crucial for economic growth and for maintaining the quality of our lives. But current levels of energy consumption and production are not sustainable. About 40% of the world's energy comes from oil, and much of that goes to transportation uses. Oil prices are dependent as much upon new (or foreseen) discoveries as they are upon political events and situations around the world. The U.S., with 4.5% of the world's population, consumes 24% of the world's oil production per year; 66% of that oil is imported!

Renewable and Nonrenewable Energy Sources

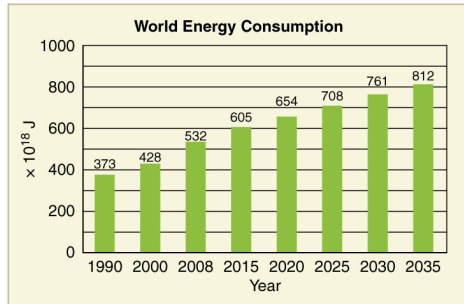
The principal energy resources used in the world are shown in [\[link\]](#). The fuel mix has changed over the years but now is dominated by oil, although natural gas and solar contributions are increasing. **Renewable forms of energy** are those sources that cannot be used up, such as water, wind, solar, and biomass. About 85% of our energy comes from nonrenewable **fossil fuels**—oil, natural gas, coal. The likelihood of a link between global warming and fossil fuel use, with its production of carbon dioxide through combustion, has made, in the eyes of many scientists, a shift to non-fossil fuels of utmost importance—but it will not be easy.



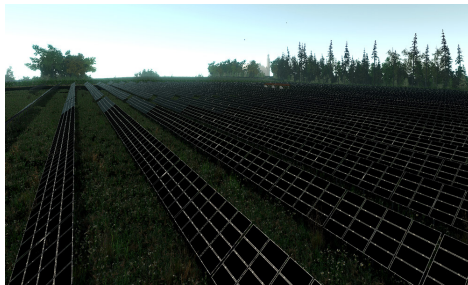
World energy consumption by source, in billions of kilowatt-hours: 2006. (credit: KVDP)

The World's Growing Energy Needs

World energy consumption continues to rise, especially in the developing countries. (See [\[link\]](#).) Global demand for energy has tripled in the past 50 years and might triple again in the next 30 years. While much of this growth will come from the rapidly booming economies of China and India, many of the developed countries, especially those in Europe, are hoping to meet their energy needs by expanding the use of renewable sources. Although presently only a small percentage, renewable energy is growing very fast, especially wind energy. For example, Germany plans to meet 20% of its electricity and 10% of its overall energy needs with renewable resources by the year 2020. (See [\[link\]](#).) Energy is a key constraint in the rapid economic growth of China and India. In 2003, China surpassed Japan as the world's second largest consumer of oil. However, over 1/3 of this is imported. Unlike most Western countries, coal dominates the commercial energy resources of China, accounting for 2/3 of its energy consumption. In 2009 China surpassed the United States as the largest generator of CO₂. In India, the main energy resources are biomass (wood and dung) and coal. Half of India's oil is imported. About 70% of India's electricity is generated by highly polluting coal. Yet there are sizeable strides being made in renewable energy. India has a rapidly growing wind energy base, and it has the largest solar cooking program in the world.



Past and projected world energy use
(source: Based on data from U.S.
Energy Information Administration,
2011)



Solar cell arrays at a power plant in
Steindorf, Germany (credit: Michael
Betke, Flickr)

[\[link\]](#) displays the 2006 commercial energy mix by country for some of the prime energy users in the world. While non-renewable sources dominate, some countries get a sizeable percentage of their electricity from renewable resources. For example, about 67% of New Zealand’s electricity demand is met by hydroelectric. Only 10% of the U.S. electricity is generated by renewable resources, primarily hydroelectric. It is difficult to determine total contributions of renewable energy in some countries with a large rural population, so these percentages in this table are left blank.

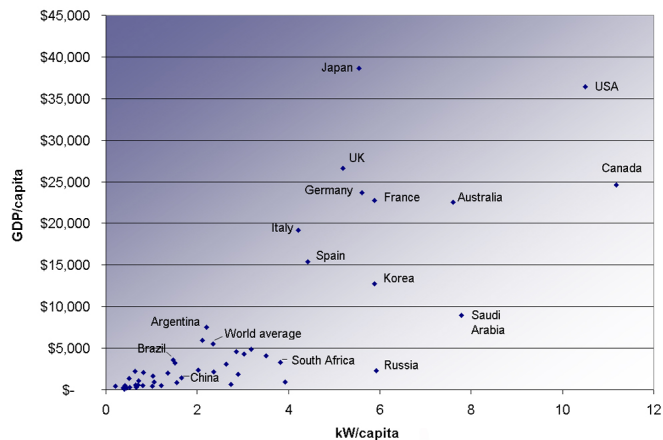
Country	Consumption, in EJ (10 ¹⁸ J)	Oil	Natural Gas	Coal	Nuclear	Hydro	Other Renewables
Australia	5.4	34%	17%	44%	0%	3%	1%

Country	Consumption, in EJ (10^{18} J)	Oil	Natural Gas	Coal	Nuclear	Hydro	Other Renewables
Brazil	9.6	48%	7%	5%	1%	35%	2%
China	63	22%	3%	69%	1%	6%	
Egypt	2.4	50%	41%	1%	0%	6%	
Germany	16	37%	24%	24%	11%	1%	3%
India	15	34%	7%	52%	1%	5%	
Indonesia	4.9	51%	26%	16%	0%	2%	3%
Japan	24	48%	14%	21%	12%	4%	1%
New Zealand	0.44	32%	26%	6%	0%	11%	19%
Russia	31	19%	53%	16%	5%	6%	
U.S.	105	40%	23%	22%	8%	3%	1%
World	432	39%	23%	24%	6%	6%	2%

Energy Consumption—Selected Countries (2006)

Energy and Economic Well-being

The last two columns in this table examine the energy and electricity use per capita. Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (gross domestic product) per capita, are matched by higher levels of energy consumption per capita. This is borne out in [\[link\]](#). Increased efficiency of energy use will change this dependency. A global problem is balancing energy resource development against the harmful effects upon the environment in its extraction and use.



Power consumption per capita versus GDP per capita for various countries. Note the increase in energy usage with increasing GDP. (2007, credit: Frank van Mierlo, Wikimedia Commons)

Conserving Energy

As we finish this chapter on energy and work, it is relevant to draw some distinctions between two sometimes misunderstood terms in the area of energy use. As has been mentioned elsewhere, the “law of the conservation of energy” is a very useful principle in analyzing physical processes. It is a statement that cannot be proven from basic principles, but is a very good bookkeeping device, and no exceptions have ever been found. It states that the total amount of energy in an isolated system will always remain constant. Related to this principle, but remarkably different from it, is the important philosophy of energy conservation. This concept has to do with seeking to decrease the amount of energy used by an individual or group through (1) reduced activities (e.g., turning down thermostats, driving fewer kilometers) and/or (2) increasing conversion efficiencies in the performance of a particular task—such as developing and using more efficient room heaters, cars that have greater miles-per-gallon ratings, energy-efficient compact fluorescent lights, etc.

Since energy in an isolated system is not destroyed or created or generated, one might wonder why we need to be concerned about our energy resources, since energy is a conserved quantity. The problem is that the final result of most energy transformations is waste heat transfer to the environment and conversion to energy forms no longer useful for doing work. To state it in another way, the potential for energy to produce useful work has been “degraded” in the energy transformation. (This will be discussed in more detail in [Thermodynamics](#).)

Section Summary

- The relative use of different fuels to provide energy has changed over the years, but fuel use is currently dominated by oil, although natural gas and solar contributions are increasing.
- Although non-renewable sources dominate, some countries meet a sizeable percentage of their electricity needs from renewable resources.
- The United States obtains only about 10% of its energy from renewable sources, mostly hydroelectric power.
- Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (Gross Domestic Product) per capita, are matched by higher levels of energy consumption per capita.
- Even though, in accordance with the law of conservation of energy, energy can never be created or destroyed, energy that can be used to do work is always partly converted to less useful forms, such as waste heat to the environment, in all of our uses of energy for practical purposes.

Glossary

renewable forms of energy

those sources that cannot be used up, such as water, wind, solar, and biomass

fossil fuels

oil, natural gas, and coal